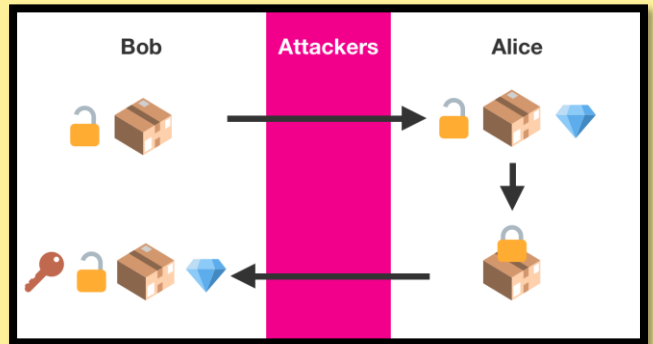# A cats' story – a boring detective story

EULER TOTIENT FUNCTION : φ (n)

- From $\phi(5)$ and $\phi(7)$, $\phi(n)$ will be n-1 whenever n is a prime number.
- This implies that $\phi(n)$ will be easy to calculate when n has exactly two different prime factors:

$$\phi(P * Q) = (P\text{-}1)*(Q\text{-}1)$$

if P and Q are prime.

Bob   Attackers   Alice

Our characters

Mr. Lee

Mrs. Lee

Ken - the lawyer

Angela - the private investigator

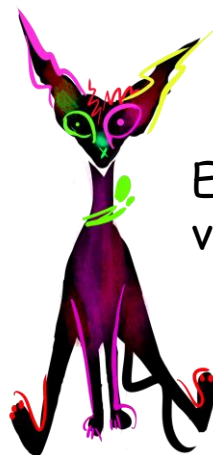Sunny - the bad son

Joe - the good son

Mary - the good son's wife

Our characters

Matt – the thug

Natalie – the investment bank IT specialist

Eva - the vice squad

The story unfolds

It is late. Angela is debating with herself whether to go home to watch an episode of the 2nd season of Lincoln Lawyer on Netflix; or to join a friend in a nearby pub for a cocktail or just flips over some old files. Business is lean in the summer. All that she had was to locate missing parrots, dogs, etc. It seems in the summer; unfaithful husbands have better thing to do than to chase after other women. So, less marital affairs to handle. These are the creams of her business.

It is then the phone rings. The caller is a lawyer whom she met a couple of months ago in her auntie's place. Although they went to the same university in the United Kingdom, it was in different epoch, and he is much older than Angela.

How's business? I need a good investigator over a small bother.

Business is not great. I have seen better days. Anyway, I am available and what can I do to help? But my fee is $3,000 per day plus expenses.

That's alright. I can live with that. From what I heard about you; you are the ideal person for the job.

So, shoot. What can I do for you?

About a week ago, I was bothered by a thug. He first called me on the phone. When I ignored his calls, he cornered me on the street. He is becoming more and more aggressive, though.

Are you intimidated? Did you call the police?

Not at all. In my generation, a lot of people like me grew up in low-cost housing estates. We are no stranger to harassment, mugging and petty thefts. No, I am not intimidated.

It is not necessary for now to go to the police. In fact, what I need is someone who can find out why and do some background digging for me. This little thug does not bother me at all. I can handle it.

What does he want from you?

He wanted me to hand over some documents that were entrusted to me by a late Mr. Lee about 3 years ago. Mr. Lee died a few months afterward. He was a mathematician and worked for many years in a French investment bank. He left behind his wife and 2 grown up sons.

Mr. Lee got terminal cancer and he knew that he did not have long to live. He was then almost 65. His wife is senile but in good health.

He seems very well off.

He told me that he set up a bank account in the Maritime Bank. This account will pay a fixed amount to his wife every month for her upkeeping. It is a very generous allowance. But if she ever needs more money, she can log into the account with a password given to her. But for each of these transactions, she cannot withdraw more than $30,000 and she is limited to two transactions per month.

Then, he contacted me.

He told me that he had left behind a fortune in a bank account in the Maritime Bank which is the sum I mentioned earlier. As long as Mrs. Lee lives, there is enough money to keep her going and much more. He left me with 2 letters; both are sealed; one of which is an instruction and is to be opened only when Mrs. Lee passes away.

He told me that he had also left a letter with each of his sons. When their mother passes away, they are to contact me and to bring along those letters. Then, I am to open the instruction letter before everyone and follow the instruction on it.

The thug wanted me to hand over to him the two letters given to me by Mr. Lee. I flatly refused and told him to go to hell. I said if he comes back again, I will call the police.

But as a precaution, all the same, I have those 2 documents safely tucked away in a safe in my office.

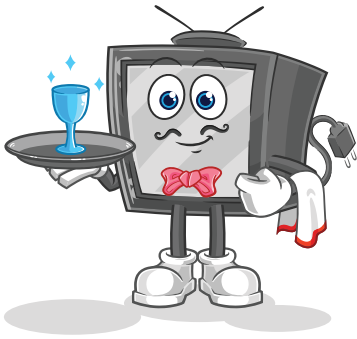How did that thug contact you?

He first called me with a mobile number. Here it is.

Can you describe to me what he looks like?

Can I also have the contact details of Mrs. Lee and his sons? Please let them know that they will be contacted by me within the next few days.

Alright, Ken. Give me a couple of weeks, I will come back to you as soon as I can.

It was too late for anything else. So, she went home. Make herself a simple meal and ate it while watching an episode of Lincoln Lawyer. It seems Mickey Haller has much more interesting cases than Ken.

Still, she has an intriguing one in her portfolio and a paying client; and it is certainly more interesting than looking for stray parrots and dogs in parks and alleys.
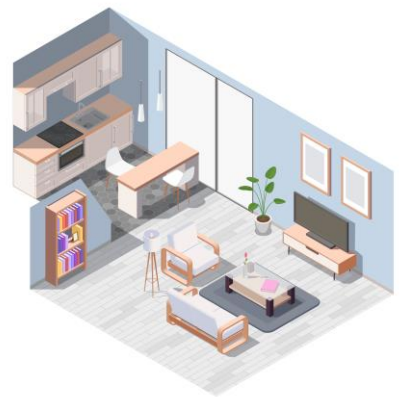
On the next day, she called Mrs. Lee. They agreed to meet in Mrs. Lee's home.

She was received by two women: Mrs. Lee and her daughter in law. Her husband is the elder son of Mrs. Lee. His name is Joe. She is called Mary.

Mary tells Angela that Mrs. Lee is senile. She needs help from time to time, especially when she needs to take some money out of Mrs. Lee's account.

Angela says that she is told that Mrs. Lee has a generous living allowance. Why would she still withdraw from the bank every now and then?

Mary takes Angela aside and out of earshot of Mrs. Lee. She says that Sunny, the brother of Joe comes to Mrs. Lee often for money. Not only did Mrs. Lee gives away to him a big chunk of her allowance, but she also withdrew each month $60,000 to give to Sunny.

Why does he come to his mother for money?

Sunny got himself mixed up with bad company. He runs up gambling debt. He even asks Joe for money from time to time. I stop seeing Sunny now.
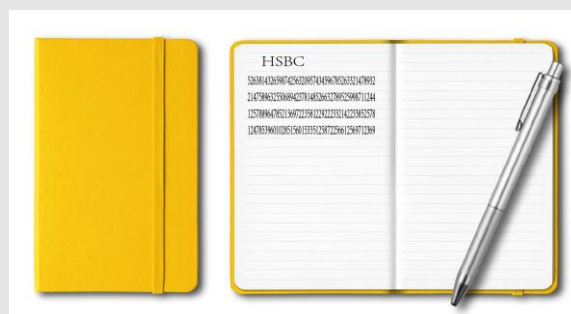
Why does Mrs. Lee need you to withdraw money for her?

Mrs. Lee is a highly educated woman. But she is a bit senile now. Although she can surf the web herself, she has difficulty with the password of Mrs. Lee's account.

What? What kind of difficulty?

The password is more than 60 integers long. My mother-in-law was given a code book by her late husband. I have to input the password into the bank account for her. I can remember that it begins with 5263814 … and ends with 125879. So, I need her to give me her code book every time.

Oh! Why does Mr. Lee create such an awkward password?

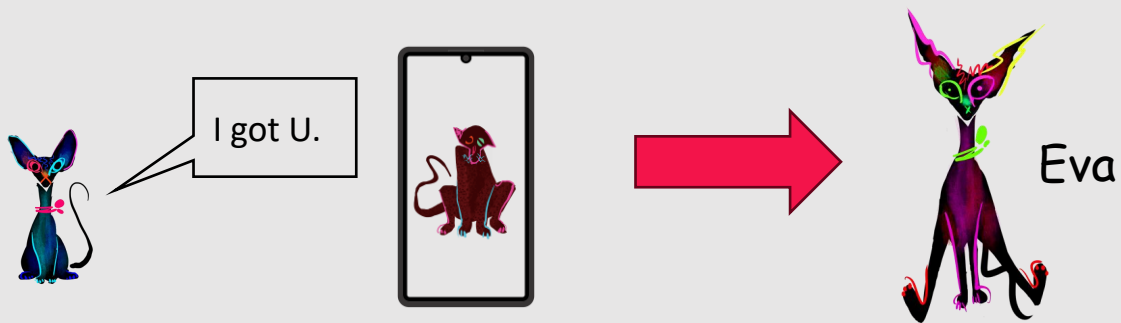Could I meet with your husband? Is he available now for a short meeting?

Angela went to the office of Joe. He is an architect and a very cordial person like his mother. But he is very evasive on the subject of his brother and does not give away too much information on Sunny. He tells Angela that at this hour, he can probably find Sunny in a bar called Sunset Boulevard.

It turns out that the bar is within walking distance from the office of Joe.

With the picture of Joe given to her by Mrs. Lee, Angela found Joe in a bar nurturing a drink. Angela keeps her distance in a far corner of the bar. It is still early and there are not a lot of people in the bar. She orders a drink as well. After about half an hour, a man approaches Joe and taps him on the shoulder. This man looks very much like the man described to her by Ken. She dials the mobile number given to her by Ken. That man's phone rings.

So, that's the man.

I got U.

Eva

With her handphone, she takes a photo of the man and Sunny.

It is still early in the day. Angela decides to call up one of her contacts in the vice squad who happens to be her sister.

"Could I send you a couple of photos and could you see whether they are someone you can recognize? Thanks very much."

Her sister comes back within 20 minutes. The man is a notorious thug and has served prison terms for extortion, etc. His name is Matt. He is up to no good. She has nothing on file on Sunny, though.

Angela returns to her office. She hasn't finished her drink.

She sits and thinks. So far, she has a lot of information that does not tie up, many loose ends.

1.   Sunny and Matt are related. This is sure.

2.   Is he using the thug or is the thug using him?

3.   Sunny must have owed the thug money. This is why he is always short. This is why he has to extort money from his mother every month.

4.   Why does the thug want the two documents given to Ken by Mr. Lee? It can only be something to do with money.

5.   So, it is possible that Sunny has said something to the thug about the documents. He may have even given or shown to the thug the letter given to him by his father.

6.   So, the document is the key to the question.

Maybe, just maybe, Joe knows something about the document. Before she picks up the phone to call Joe, Mary calls.

After discussing with you today, Joe would like you to know that under duress from his brother, Joe has handed over his father's letter to Sunny. But Joe has no idea what is on the letter.
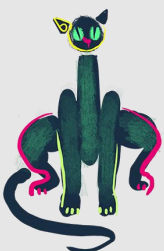
???

Things are becoming more and more curious.

Obviously, the letters are important. It is highly probable that Sunny and Matt know the contents of these letters. This is why Matt wants to have the letters from Ken. With Ken's documents, Matt and Joe will know exactly what is in waiting for Joe when his mother dies.

Together with her sister, they visit Sunset Boulevard. It is amazing that Sunny is still somewhat sober after all these hours.

Matt was nowhere to be found on the premises.

They put questions to Sunny. At first, he refuses to answer. But he loosens up when Angela threatens to bring his mother to the scene.

I don't mind telling you. I don't know what it is. I don't know what my father is thinking. On the letter to Joe and me, there is nothing but a string of 20 random numbers. For me, it is something like

45278… 8970. I don't remember exactly.

On the one of Joe, it is something like 2541208 …33312.

It makes no sense

Neither for Angela nor her sister.

They leave Sunny to his misery.

Angela returns to her office.

A big question mark hangs over her. Why?

Now, the letters that Ken has in his possession. One is an instruction: to be opened only before Joe and Sunny after their mother dies.

It is logical that when she dies, all the money left behind is to be shared between the brothers or given to charity, etc.

So, what's in the second letter given to Ken?

Following the logic of the first two letters, the letter may contain inside another 20 random numbers.

So, we have 3 sets of random numbers? And for what?

Then, she remembers what Mary told her in the morning that the password to Mr. Lee's bank account is also made up of many integers.

So, what are these numbers and how are they related?

Her head starts to ache. She decides to take a break.

During the walk, another thing strikes her. Mr. Lee is a mathematician. So, his work is related to numbers. This is clear.

OMG, more numbers!

I am going nowhere.

Maybe Mrs. Lee can help by shedding some light on the job of his husband.

Too late to call. Tomorrow morning then.

Mrs. Lee, what does your husband do in the French investment bank?

He does not talk to me about his work. All I know is he works in the computer department of this investment bank.
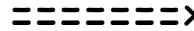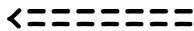
I think Sunny is in big trouble. I want to help. Is there someone I can speak to in the investment bank about your husband's work?

Mr. Lee is highly respected in the bank, and it seems he is a mentor to a lot of young people in the IT department. One of them calls Natalie still calls me from time to time to say hello. Here is her number. I will ring her up to let her know that you will call.

In the afternoon, Angela calls Natalie. Natalie is expecting her call. Regarding Mr. Lee, Natalie is more than amiable.

<===== =======>

Mr. Lee is our head cryptographer.

What is that? What does he do? Let me be clear. I am not a friend of Mrs. Lee. I am a private investigator working for a lawyer who has been entrusted with some papers by Mr. Lee before he passed away. It seems these documents are so important that someone would like to snatch them from my client. It seems also harms maybe done to Mrs. Lee and his family if we don't do something about it and to get to the bottom of it.

Maybe it is better to come to my office and to talk over a coffee. OK with you?

Like most IT professionals, Natalie is very fair skinned and pale; not enough sun, obviously.

OK. The main job of a cryptographer has to do with encryption and decryption. Looking at your face, I am probably jumping ahead of myself. I will start with coding.

"Say, suppose I want to send an email to my lover during my office hour to tell him 'I love him'. This message may be seen by someone in the office. This is not good for me nor for him. So, I coded the message and then with a pre-agreed procedure, he will decode the message and see what I really wanted him to see."

The simplest one is what we called Caesar cipher or shift cipher. A trick attributed to Julius Caesar.

I shifted every letter alphabetically to the right. If it is shifted by two letters, I have then

I	L	O	V	E	U
K	N	Q	X	G	W

Then, I will write in my email KNQXGW. My friend James will then shift my message reversely by shifting every letter to the left twice.

In this case, I am the cryptographer. I encrypted the message; and James decrypted the message.

OK?

Then, there are other methods of sending messages like the smoke signals by the Chinese on the Great Wall to forewarn invasion by foreign tribes. The American Indians have the same thing.

The system developed and became more and more sophisticated over the years. The most sophisticated one is perhaps the so-called Enigma code machine *(see the photo on page 1)* used by the German during World War II. It took 10,000 people + Alan Turing + the first computer in the world and a bit of luck to finally decode messages sent by the Enigma machines. It may have shortened the great war by 2 years.

But in all the above systems, both the sending party and the receiving party know the code procedure.

Now, how would two perfect strangers be able to exchange confidential information without pre-agreed procedure? They don't know each other. They have no reason to trust the other party. Then, what do they do?

How?

1.   Like your Gmail. Ideally, although your message is sent through Google, Google does not need to know the contents of your email. Google encrypted your message and then decrypted the message for your friend when he or she opens your message. So, you only need one "key" to see all the messages instead of one key for each of your friends.

2.   Your bank account. Your bank does not keep a copy of your login password. It keeps a hash of your password. A hash is a function in computer parlance, something to do with mathematics. With the hash, your password will be accepted. You will be given a token and a session to carry out your transactions. Since the bank does not know your password, its employees have no way to steal money from your account.

Wow. How does it work?

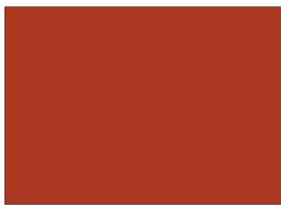I will give you two examples of this so-called one-way function.

1.   Let's take you and me. We first agree on a common colour, say green. OK? Then we each have a private colour that only you or I know. Let's say my private colour is red and yours is blue, OK?

1.1.   Then, you mix your private colour with the green. The result is a colour that nobody knows is made up of green and blue. One can guess what it is but there are several possibilities. I do the same thing. I mix the green with my red. Again, the resulting colour is something difficult to tell it is from blue + red.

1.2.   Then, you send me your mix (blue + green).

1.3.   I send you my mix (red + green).

1.4.   You add to my mix (red + green) with your blue. The mixture is thus red + green + blue.

1.5.   I add to your mix (blue + green) with red. The resulting mixture is blue + green + red.

1.6.   So, although we do not know the private colour of the other party, we got the same colour mix.
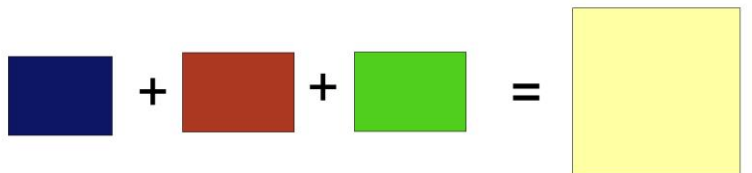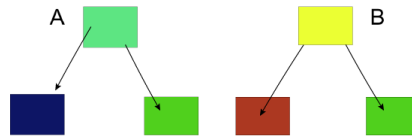
This is the trick.

private colour of Angela

private colour of Natalie

agreed public colour

common colour

A   B

it is virtually impossible to tell which
colours made up A or B

2.   Now something akin to what Mr. Lee is doing in the investment bank.

2.1.   We take 2 prime numbers. Prime numbers are like 1, 3, 5, 7, 11, 13, 17, etc.

2.2.   Let's say 2 very large prime numbers, say those with 20 integers. Let's call them P1 and P2.

2.3.   If we multiply them together, we get a very large number, say x.

2.4.   x = P1 x P2.

2.5.   It is very easily to get x.

2.6.   But given x, it is difficult to know what are P1 and P2.

2.7.   If x is a big enough number, it will take a supercomputer many months or years to find P1 and P2. If there are 60 digits, x = 10^{60}, a huge number.

2.8.   So, it is very easy to encrypt but very difficult to decrypt.

2.9.   This is then the basis of modern cryptography.

2.10.   Mr. Lee is our specialist in this field.

Thank you so much, Natalie. I think you have provided me with the answer to my puzzle.

I think it's time to pay Ken a visit and to tell him what have I found.

I think I have a pretty good idea of the whole thing.

Mr. Lee wants to make sure that Mrs. Lee will be well looked after when he is gone. So, he put all the money he left behind in a bank account. The password to the account is made up of 3 prime numbers, say P1, P2, and P3.

He enclosed each of this Prime number in a sealed envelope and sent it respectively to you, Joe, and Sunny.

So, password = P1 * P2 * P3 and is the one on the notebook of Mrs. Lee. It's unbreakable and impossible to memorise.

So, when Mrs. Lee passes away, you will each open the letter and will be told in the instruction on how the money will be split.

Then, why does he limit the monthly withdrawal to $30,000 and twice a month?

Now, Mr. Lee may think that Sunny cannot be trusted. He may steal the password codebook from his mother and withdraw all the money for himself. By limiting to $60,000 per month, Sunny may think that his mother is giving him some of her monthly allowances, which she does.

So, the code book is a secret he kept with his wife. Maybe Sunny and Joe do not even know of its existence.

Neither Sunny nor Joe knows about the codebook. Mary may have an inkling. But she probably doesn't know enough. So, when Mrs. Lee passes away, the codebook may be lost or buried with her other belongings. Since the bank does not have the password, even with a court order, it will take a long time for Sunny and Joe to get hold of the money, if, it is earmarked for them.
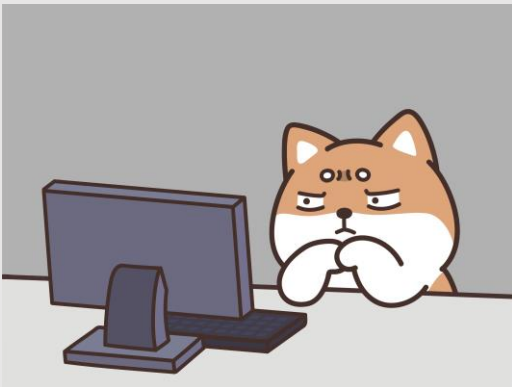
So, maybe this is in the instruction given to me … perhaps to explain everything.

So, what will you do now?

I can put the theory to a judge. It's just a theory at this stage. But since I have been threatened, it gives rise to my right to propose a resolution of the matter. Either the judge will allow me to open the letters before him in front of the Lee's or I can just do it without him and only in front of the Lee's. I would have to consult the judge.

I will let you know the outcome. But good investigative work, Angela. Bravo.

It seems a long time to wait. Finally, after 3 weeks, Ken came to see Angela with a cheque.

Everything is now resolved. I opened the letters before the Lee's, and it was as you have suspected. The money is to be split 50 – 50 between the two brothers on the demise of Mrs. Lee. I "suggest" to Sunny to tell Matt to lay off. Sunny does not have access to the fund until his mother dies. The police will be vigilant about foul-play to make sure that Mrs. Lee will live happily here after. Since Sunny knows how much he has coming, he can pacify Matt for the moment. If he makes changes to his life, to make himself a better person and to live a less decadent life, he will not have to look over his shoulders again and again for someone like Matt. But it is totally up to him.

So, everything went well. After a warm hand shake, Ken gives the cheque to Angela. It is more than Angela would have expected.

And Ken promises to recommend Angela to all his clients, peers and colleagues.

Time to look for the missing parrots and dogs again …

And no more worries for the rent due end of the month 🙂

- The end -

Eddie Shum 28.8.23